1. Cancel out common factors in the numerator and the denominator:

$$\frac{4y}{9x}\cdot\frac{15xy}{8y}=\frac{4y}{3*3\cancel{x}}\cdot\frac{3*5\cancel{x}\cancel{y}}{2*4\cancel{y}}=\frac{5y}{3}$$

2. Factor the second fraction:

$$x^2-1=(x-1)(x+1)$$
$$x^2-x-6=x^2-3x+2x-6=x(x-3)+2(x-3)=(x-3)(x+2)$$

Therefore,

$$\frac{3x+6}{x+1}\cdot\frac{x^2-1}{x^2-x-6}=\frac{3\cancel{(x+2)}}{\cancel{x+1}}\cdot\frac{(x-1)\cancel{(x+1)}}{(x-3)\cancel{(x+2)}}=\frac{3(x-1)}{x-3}$$

3. First convert the division to multiplication by flipping the second fraction, then cancel out common terms:

$$\frac{7u}{15ux}\div\frac{3u}{5x}=\frac{7u}{15ux}\cdot\frac{5x}{3u}=\frac{7\cancel{u}}{3*\cancel{5x}\cancel{u}}\cdot\frac{\cancel{5x}}{3u}=\frac{7}{9u}$$

4. $$\frac{\frac{6p^3q}{5mn^3}}{\frac{3p^5q^4}{15n^4}}=\frac{6p^3q}{5mn^3}\cdot\frac{15n^4}{3p^5q^4}=\frac{6*15}{5*3}\cdot\frac{1}{m}\cdot\frac{n^4}{n^3}\cdot\frac{p^3}{p^5}\cdot\frac{q}{q^4}=6\cdot\frac{1}{m}\cdot n\cdot\frac{1}{p^2}\cdot\frac{1}{q^3}=\frac{6n}{mp^2q^3}$$

5. First simplify the fraction:

$$\frac{2z}{2+\frac{3}{z+1}}=\frac{2z}{\frac{2(z+1)}{z+1}+\frac{3}{z+1}}=\frac{2z}{\frac{2z+2+3}{z+1}}=\frac{2z}{\frac{2z+5}{z+1}}=\frac{2z(z+1)}{2z+5}$$

Then the denominator in the original expression is

$$z+\frac{2z}{2+\frac{3}{z+1}}=z+\frac{2z\left(z+1\right)}{2z+5}=z\left(1+\frac{2z+2}{2z+5}\right)=z\left(\frac{2z+5+2z+2}{2z+5}\right)=\frac{z\left(4z+7\right)}{2z+5}$$

So

$$\frac{z}{z+\frac{2z}{2+\frac{3}{z+1}}}=\frac{\cancel{z}}{\frac{\cancel{z}\left(4z+7\right)}{2z+5}}=\frac{1}{\frac{\left(4z+7\right)}{2z+5}}=\frac{2z+5}{4z+7}$$

6.

$$\frac{5t+4u}{6t}-\frac{6t-8u}{6t}=\frac{\left(5t+4u\right)-\left(6t-8u\right)}{6t}=\frac{5t+4u-6t+8u}{6t}=\frac{\left(5t-6t\right)+\left(4u+8u\right)}{6t}=\frac{-t+12u}{6t}$$

7. Write each of the fractions using the LCD as the denominator:

$$\frac{2x+7w}{5x}+\frac{8x-7w}{2x}-3=\frac{2\left(2x+7w\right)}{10x}+\frac{5\left(8x-7w\right)}{10x}-\frac{3\cdot 10x}{10x}$$
$$=\frac{\left(4x+14w\right)+\left(40x-35w\right)-30x}{10x}=\frac{\left(4x+40x-30x\right)+\left(14w-35w\right)}{10x}$$
$$=\frac{14x-21w}{10x}=\frac{7\left(2x-3w\right)}{10x}$$

8. The LCD is $x(x-1)$, so

$$-\frac{4}{x-1}+\frac{3-x}{x}=\frac{-4x}{x\left(x-1\right)}+\frac{\left(3-x\right)\left(x-1\right)}{x\left(x-1\right)}=\frac{-4x}{x\left(x-1\right)}+\frac{3x-3-x^2+x}{x\left(x-1\right)}$$
$$=\frac{-4x+3x-3-x^2+x}{x\left(x-1\right)}=\frac{-x^2-3}{x\left(x-1\right)}=-\frac{x^2+3}{x\left(x-1\right)}$$

9. Factor the denominators

$$x^2-2x-24=\left(x-6\right)\left(x+4\right)$$
$$x^2-7x+6=\left(x-6\right)\left(x-1\right)$$

Then the LCD is $(x-6)(x-1)(x+4)$.

Write the fractions using the LCD as the denominator:

$$\frac{2}{x^2-2x-24}-\frac{1}{x^2-7x+6}=\frac{2}{(x-6)(x+4)}-\frac{1}{(x-6)(x-1)}$$
$$=\frac{2(x-1)}{(x-6)(x-1)(x+4)}-\frac{x+4}{(x-6)(x-1)(x+4)}=\frac{2x-2-x-4}{(x-6)(x-1)(x+4)}$$
$$=\frac{x-6}{(x-6)(x-1)(x+4)}=\frac{1}{(x-1)(x+4)}$$

10. Factor the polynomials then cancel out the common factors:

$$z^2-10z+24=z^2-4z-6z+24=z(z-4)-6(z-4)=(z-4)(z-6)$$
$$4z^2-4z-48=4(z^2-z-12)=4(z-4)(z+3)$$

So

$$\frac{z^2-10z+24}{4z^2-4z-48}=\frac{(z-4)(z-6)}{4(z-4)(z+3)}=\frac{z-6}{4(z+3)}$$

11. extract the common factor when factoring the numerator:

$$\frac{4t^2+20t^3w}{8t^4u^3}=\frac{4t^2(1+5tw)}{8t^4u^3}=\frac{4t^2(1+5tw)}{4t^2\cdot 2tu^3}=\frac{1+5tw}{2tu^3}$$

12. Multiply y to both sides:

$$6=-9y\Rightarrow y=-\frac{6}{9}=-\frac{2}{3}$$

13. Multiply (x+ 6) to both sides

$$8 = -3(x+6)$$
$$x+6 = -\frac{8}{3}$$
$$x = -\frac{8}{3} - 6$$
$$x = -\frac{26}{3}$$

14. Multiply the LCD (z -2)(z -1) to both sides and simplify

$$\frac{5}{z-2}(z-2)(z-1) = \left(-8 - \frac{1}{z-1}\right)(z-2)(z-1)$$
$$5(z-1) = -8(z-2)(z-1) - (z-2)$$
$$5z - 5 = -8(z^2 - 3z + 2) - z + 2$$
$$8z^2 - 24z + 16 + z - 2 + 5z - 5 = 0$$
$$8z^2 - 18z + 9 = 0$$

Solving the quadratic equation gives

$$(2z-3)(4z-3) = 0$$
$$z = \frac{3}{2} \quad or \quad z = \frac{3}{4}$$

Substituting above values back to the original equation to check it makes sense, that is, the denominators are not equal to 0.

Finally, the solutions are ¾, or 3/2.

15. Multiply (y + 3) (y-3) to both sides of equation

$$\frac{y+2}{y+3}(y+3)(y-3)=\left(\frac{y+3}{y-3}+1\right)(y+3)(y-3)$$
$$(y+2)(y-3)=(y+3)^2+(y+3)(y-3)$$
$$y^2-y-6=y^2+6y+9+y^2-9$$
$$y^2+7y+6=0$$
$$(y+1)(y+6)=0$$
$$y=-1 \quad or \quad y=-6$$

Substituting above values back to the original equation to check it makes sense, that is, the denominators are not equal to 0.

Finally, the solutions are -1, or -6.

16. using the cross product: $\frac{a}{b}=\frac{c}{d}\Rightarrow ad=bc$

We have

$$11*6=17v$$
$$v=\frac{66}{17}$$

17. Using the cross product,

$$-15(x+2)=-21x$$
$$-15x-30=-21x$$
$$-15x+21x=30$$
$$6x=30$$
$$x=5$$

Substituting above values back to the original equation to check it makes sense, that is, the denominators are not equal to 0.

Finally, the solution is 5.

18. Find the tax rate using the first item:

$$r = \frac{tax}{sales} = \frac{1800}{18900} = \frac{18}{189}$$

Then the sales tax for the second item is

$$t = sales * rate = 52.50 * \frac{18}{189} = 5 \text{ dollars.}$$

19. The area of the triangle is

$$A = \frac{1}{2} base * height$$

$$base = \frac{2A}{height} = \frac{2\left(x^2 + 5x + 6\right)}{x+3} = \frac{2(x+3)(x+2)}{x+3} = 2(x+2) \text{ meters}$$