Question (1):

Find the expected values of the following probability distributions.

(a)

Y_i	$p(Y_i)$
5	0.15
10	0.25
15	0.05
20	0.20
25	0.35

(b)

Y_i	$p(Y_i)$
5	0.20
10	0.30
15	0.15
20	0.30
25	0.05

Solution (a):

Expected Value = $\sum Y_i p(Y_i)$

= 5(0.15) + 10(0.25) + 15(0.05) + 20(0.20) + 25(0.35)

= 0.75 + 2.5 + 0.75 + 4 + 8.75

= **16.75** **[Answer]**

Solution (b):

Expected Value = $\sum Y_i p(Y_i)$

= 5(0.20) + 10(0.30) + 15(0.15) + 20(0.30) + 25(0.05)

= 1 + 3 + 2.25 + 6 + 1.25

= **13.5** **[Answer]**

Question (2)

In a population of test scores not known to be normally distributed, the mean is 50 and the standard deviation is 5. What proportion of the observations will fall in the interval from 40 to 60?

Solution: Given $\mu = 50$, $\sigma = 5$, a = 40 and b = 60

We can apply the central limit theorem.

$$P(60 < \bar{X} < 40) \approx P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right)$$

$$\approx P\left(\frac{40-50}{5} < Z < \frac{60-50}{5}\right)$$

$$= P(-2 < Z < 2)$$

= 2 (0.4772) = 0.9544   [0.4772 is the value of z for 2 from tables]

Therefore, $P(60 < \bar{X} < 40)$ = **0.9544 or 95.44%   [Answer]**

Question (3):
Using the central limit theorem, find the means and standard errors of sampling distributions with the following characteristics:

	μ_Y	σ_Y^2	N
a.	10.5	50	25
b.	50	11.5	11.5
c.	25	75	250
d.	12	70	35
e.	100	100	200

Solution (a):
From central limit theorem, for all populations we have

Mean $\mu = \mu_Y$, Standard error $\sigma_Y = \frac{\sigma_Y}{\sqrt{n}}$

Therefore, $\mu = \mu_Y$ = 10.5

Standard error $\sigma_Y = \frac{\sigma_Y}{\sqrt{n}} = \frac{\sqrt{50}}{\sqrt{25}} = \frac{5\sqrt{2}}{5} = \sqrt{2}$ = 1.414

Solution (b):

$\mu = \mu_Y$ = 50

Standard error $\sigma_Y = \frac{\sigma_Y}{\sqrt{n}} = \frac{\sqrt{115}}{\sqrt{115}} = 1$

Solution (c):

$\mu = \mu_Y$ = 25

Standard error $\sigma_Y = \frac{\sigma_Y}{\sqrt{n}} = \frac{\sqrt{75}}{\sqrt{250}} = \frac{5\sqrt{3}}{5\sqrt{10}} = \frac{\sqrt{3}}{\sqrt{10}} = 0.5477$

Solution (d):

$\mu = \mu_Y$ = 12

$$\sigma_Y = \frac{\sigma_Y}{\sqrt{n}} = \frac{\sqrt{70}}{\sqrt{35}} = \frac{\sqrt{35}\sqrt{2}}{\sqrt{35}} = \sqrt{2} = 1.414$$

Solution (e):

$\mu = \mu_Y$ = 100

Standard error = $\sigma_Y = \frac{\sigma_Y}{\sqrt{n}} = \frac{\sqrt{100}}{\sqrt{200}} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$ = 0.7071

Question (4)

Test the null hypothesis that $\mu_Y = 75$ for a sample of 25 subjects in which $\overline{Y} = 81$ And $S_Y = 10$ for a one-tailed test in which $\alpha = 0.01$. State (a) critical value (b) degrees of freedom (c) test statistic and (d) your decision.

Solution:

Since the sample size is small, (that is less than 30), we use t-statistic.

Null Hypothesis H_0: $\mu_Y = 75$

Alternate Hypothesis H_1: $\mu_Y > 75$

(a) Critical value of t-statistic for 24 degrees of freedom = 2.492

(b) Degrees of freedom = n – 1 = 25 – 1 = 24

(c) Test statistic = $t = \dfrac{\overline{Y} - \mu_Y}{\dfrac{S_Y}{\sqrt{n}}} = \dfrac{81 - 75}{\dfrac{10}{\sqrt{25}}} = \dfrac{6}{2} = 3$

(d) Since the test statistic 3 > 2.492 (critical value), we reject the Null Hypothesis H_0 and accept H_1.

Thus, at $\alpha = 0.01$ level we conclude that $\mu_Y > 75$

Question (5):

Test the Null Hypothesis that $\mu_Y = 50$ for a sample where N = 36, $\overline{Y} = 47.5$
And $S_Y = 12$ using a two-tailed test in which $\alpha = 0.001$. State (a) Critical value
(b) Degrees of freedom (c) test statistic (d) your decision.

Solution:

Since n = 36 > 30, we use z-statistic to test the hypothesis.

Null Hypothesis H_0: $\mu_Y = 50$

Alternate Hypothesis H_1: $\mu_Y \neq 50$

(a) Critical value of z-statistic (for $\alpha = 0.001$) = 3.27

(b) Degrees of freedom = n – 1 = 36 – 1 = 35

(c) z-statistic = $\dfrac{\overline{Y} - \mu_Y}{\dfrac{S}{\sqrt{n}}} = \dfrac{47.5 - 50}{\dfrac{12}{\sqrt{36}}} = \dfrac{-2.5}{2} = -1.25$

(d) Since z = – 1.25 > – 3.27, we accept H_0 and reject H_1

Thus, at $\alpha = 0.001$ level we conclude that $\mu_Y = 50$.

Question (6):

After six months in office, a U.S. senator evaluates her public approval rating by surveying 50 citizens. Their responses indicate that the senator has a 63% mean approval rating with a variance of 16. If 60% is the minimum approval rating the senator will accept in order to continue her current agenda. Can she conclude that her approval rating is significantly above the minimum? Set $\alpha = 0.01$.

Solution:

Given n = 50, $\overline{X} = np = 63\% = 0.63$

Variance = npq = 16, therefore, Standard deviation = $s = \sqrt{16} = 4$

Null Hypothesis H_0: $\mu = 0.60$

Alternate Hypothesis H_1: $\mu > 0.60$

Critical value of z (for $\alpha = 0.01$) = 2.33

Z-statistic = $\dfrac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{63 - 60}{\frac{4}{\sqrt{50}}} = \dfrac{3}{0.5657}$ = 5.303

Decision: Since the computed z-statistic 5.303 > 2.33, we reject the Null Hypothesis H_0

Thus, at $\alpha = 0.01$ level (that is at 99% confidence) we conclude that her approval rating is significantly above the minimum of 60%.